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FINAL STATE INTERACTION IN HEAVY HADRON DECAY

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I present a critical account of the final-state interaction (FSI) in two-body B decays from viewpoint of the hadron picture. I emphasize that the phase and the magnitude of decay amplitude are related to each other by a dispersion relation. In a model phase of FSI motivated by experiment, I illustrate how much the magnitude of amplitude can deviate from its factorization value by the FSI.

1   Does the FSI phase diminish with the initial hadron mass ?

For a limited number of decay modes, we can extract the FSI phase directly from experiment. It is large for the D decays[1] and small for the B decays:[2]

$$\Delta\delta = 80^\circ \pm 7^\circ; \ \ D \to K^-\pi^+/\overline{K}^0\pi^+, \tag{1}$$

$$\Delta\delta < \begin{cases} 11^\circ; B \to D^+\pi^-/D^0\pi^- \\ 16^\circ; B \to D^+\rho^-/D^0\rho^- \\ 29^\circ; B \to D^{*+}\pi^-/D^{*0}\pi^-. \end{cases} \tag{2}$$

Though nothing is known for $B \to \pi\pi, K\pi, K\overline{K}$, we might conjecture that the FSI phases diminish as the initial mass increases. Meanwhile the relative FSI phase is known for the ggg and "γ" decay amplitudes of the J/ψ decays:[3] $\Delta\delta \approx 90^\circ$ and 75° for $J/\psi \to 0^-0^-$ and 0^-1^-, respectively. Therefore, if the FSI phases diminish with the increasing initial mass, it should start diminishing somewhere between $m_{J/\psi}$ and m_B or at higher energies.

Can we argue that short-distance (SD) QCD should dominate[4] in the FSI of B meson decays ? I am skeptical. Take the spectator decay $B^+(\overline{b}u) \to K^0(\overline{s}d)\pi^+(\overline{d}u)$. K^0 is formed with energetic $\overline{s}$ and d flying away colinearly, while $\overline{d}$ picks up the soft spectator u to form π^+. The gluons exchanged between $\overline{d}$ and $\overline{s}$ (or d) are hard. But the gluons between the soft spectator u and $\overline{s}$ (or d) are not so hard. The CM energy of the u and $\overline{s}$ (or d) grows only with $\sqrt{m_b}$, not linearly in m_b. By simple kinematics, $m_{\overline{d}u} \approx (\Lambda_{\rm QCD}m_b)^{1/2} \simeq 1.2$ GeV for $E_u \simeq \Lambda_{\rm QCD}$, which is in the middle of the resonance region of the $\overline{s}d$

channel. Then long-distance (LD) interactions cannot be ignored between K^0 and π^+. One may stick to the quark picture by invoking the local quark-hadron duality[5] While it may be appropriate for inclusive decays, the duality is a dynamical hypothesis yet to be tested in two-body B decays.

2   Why is the FSI complicated in the B decay ?

The FSI phases of $K \to \pi\pi$ and the hyperon decay are equal to the strong interaction phase shifts by the phase theorem, since the final states are the eigenstates of S matrix. In contrast, the FSI phases of the B decay depend on both strong and weak interactions. Take the decay $B \to K\pi$. Since $K\pi$ is made of a large number of eigenchannels a of S matrix, $|K\pi^{out}\rangle = \sum_a c_a |a^{out}\rangle$, the decay amplitude through the operator $\mathcal{O}_i$ acquires an eigenphase δ_a from each eigenchannel as

$$M^{(i)}(B \to K\pi) = \sum_a c_a M_a^{(i)} e^{i\delta_a}, \quad (c_a^* = c_a; M_a^{(i)*} = M_a^{(i)}). \qquad (3)$$

The net phase of $M^{(i)}$ depends on $\mathcal{O}_i$ even for the same isospin eigenstate of $K\pi$. The FSI phase has nothing to do with the phase shift of elastic $K\pi$ partial-wave amplitude, $\eta e^{i\delta_{K\pi}} \sin \delta_{K\pi} \equiv (\sum_a |c_a|^2 e^{2i\delta_a} - 1)/2i$ at the B mass.

3   Meson-meson interaction at $E_{\mathrm{cm}} \simeq 5$ GeV

To understand the FSI, we need to know how two mesons interact at $E_{\mathrm{cm}} = m_B$. The elastic meson-meson scattering amplitude can be written as

$$T(s,t) = i\sigma_{\mathrm{tot}} s e^{bt} + \sum_i \gamma_i(t) s^{\alpha_i(t)} + \cdots. \qquad (4)$$

The dominant term is the Pomeron exchange, which is purely imaginary and sharply peaked in the forward direction. With a bit of theory,[6] we find $\sigma_{\mathrm{tot}} = 15 \sim 20$mb and $b \approx 2.8$GeV^{-2} for $K\pi$. These values are much the same for other meson scatterings. The non-Pomeron terms are down by $\sqrt{s}$ at $t = 0$.

The partial-wave projection of $T(s,t)$ gives $a_0(m_B) \approx 0.17i$. It leads to a large inelasticity:

$$\sigma_{\mathrm{inel}}/\sigma_{\mathrm{tot}} = 1 - \mathrm{Im} a_0 \approx 0.83. \qquad (5)$$

A large portion of inelasticity presumably goes into multibody channels.

What does this imply in the two-body B decays? For the factorization-allowed processes, the intermediate states would be similar to those which appear in elastic meson scattering. Among two-body intermediate states, those

connected to the final state by the Pomeron exchange are the most important. For the factorization-forbidden processes in which a quark-pair annihilation must occur, the intermediate states of the Pomeron-exchange type are forbidden. Since the composition of the intermediate states are completely different, the factorization-allowed and forbidden amplitudes generally have quite different FSI phases even for the same isospin state of, say, $K\pi$.

4   Two-body intermediate approximation

In computing the FSI phases, one makes the two-body approximation for the intermediate states.[7] It is a dubious approximation when inelasticity of scattering is high. Getting a number even in this approximation is a daunting task, if one tries to do it right.

One starts with the unitarity sum of the absorptive part of decay amplitude for $B \to n$ (two-body) and keep only the two-body intermediate states:

$$\mathrm{Im}M_n = \sum_{n'} a_0(s)_{nn'} M_{n'}^*, \quad (s = m_B^2; n' = \text{two-body}) \tag{6}$$

where $a_0(s)_{nn'}(= a_0(s)_{n'n})$ is the $J = 0$ amplitude for $n \to n'$. Let us consider only the factorization-allowed processes. The intermediate states connected to n by Pomeron exchange dominate in the sum. Therefor all $a_0(s)_{nn'}$ are purely imaginary. Then we can easily show that M_n are also all purely imaginary. This is a special case where the FSI phases do not depend on $\mathcal{O}_i$.

Phase differences of M_n arise from the non-Pomeron exchanges, ρ, a_2, $\cdots$. Computing the non-Pomeron contributions in Eq.(6) is nearly impossible: First of all, the individual terms in the unitarity sum are not real. Only the sum total is. One has to know the phases of $M_{n'}$. This is circular. To cut the circle, one might approximate $M_{n'}$ with the factorization amplitudes. That is the Born approximation valid only for small phases. After obtaining $\mathrm{Im}M_n$, one has to compute $\mathrm{Re}M_n$ by a dispersion integral of $\mathrm{Im}M_n$ over all energies. Even if one could compute for all two-body intermediate states, one has no idea as to how much the multibody states modify the answer.

5   Random approximation

Computing the FSI phases is difficult because there are so many intermediate states that we know very little about. Can we take advantage of the presence of many states? A statistical approach or a random approximation comes to our mind. Such an attempt was actually made with a limited success.[6]

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Start with the unitarity sum, this time including all intermediate states:

$$\mathrm{Im} M_n = \sum_{n'} a_0(s)_{nn'} M_{n'}^*, \quad (s = m_B^2; n' = \mathrm{all}). \tag{7}$$

Each term in the sum is a product of the pure strong-interaction amplitude and the weak decay amplitude. Let us postulate that the sign of their product is random from one intermediate state to another when there are very many states. This is a dynamical hypothesis. It is not true if SD dominates. Under this postulate one can compute the statistically likely values of the FSI phases as their standard deviations from zero. The likely value $\overline{\delta}$ of the FSI phase depends on the "favoredness" parameter ρ. It parametrizes how much favored or disfavored the decay modes are for a given decay operator: $\rho < 1$ for the favored processes such as the factorization-allowed and color-favored modes, while $\rho < 1$ for the unfavored processes such as factorization-forbidden and/or color-suppressed modes. The relation between $\overline{\delta}_n$ and ρ is given by

$$\tan^2 \overline{\delta}_n = \frac{\tau^2(\rho^2 - \tau^2)}{1 - \rho^2\tau^2}, \quad (\tau < \rho < \frac{1}{\tau}) \tag{8}$$

where τ^2 is a number determined by elasticity of the final-state rescattering ($\tau^2 \approx 0.20$ for $K\pi$). The likely phase $|\overline{\delta}_n|$ of M_n is plotted in Fig.1. It is small for the favored modes but can be large for unfavored modes. For the "typical processes" of $\rho = 1$, the value is $\simeq 20°$. It means that the FSI phases of the typical processes are most likely between $-20°$ and $20°$.
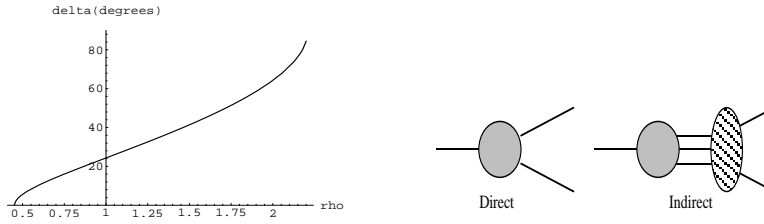

Figure 1: The likely values of the FSI phase *vs* the favoredness parameter ρ.

This trend of $\overline{\delta}$ can be understood as follows: For the favored modes, the direct decay dominates over the indirect decay going through on-shell intermediate states. Since the latter is the origin of the FSI phase, the FSI phase is small. For the disfavored modes, the indirect decays dominate since the direct decay is suppressed. The random approximation quantifies this feature in terms of the favoredness parameter ρ, given elasticity of FSI. A similar qualitative reasoning was made by Rosner[8] in the quark picture.

The limitation of the random approximation is that we cannot predict values of individual phases, but only their likely values.

6 Factorization and enhancement by FSI

6.1 Enhancement by FSI

The FSI not only generates a strong phase but also changes a magnitude of amplitude. Fermi[9] argued in potential scattering that the FSI should modify the amplitude by the wave function at origin $\varphi(0, k)$ of final two particles:

$$M \rightarrow \varphi(0, k)M. \tag{9}$$

For slow final particles, the attractive force enhances the decay by pulling them close together while the repulsive force suppresses by pushing them away. Since $\varphi(0, k)$ is equal to the Jost function $1/f(k)^*$, it also proves the phase theorem.

This theory can be extended to relativistic rescattering. In relativistic particle physics, we use the phase-amplitude dispersion relation generically referred to as the Omnès-Mushkelishvili representation[10]:

$$M(s + i\epsilon) = P(s) \exp\left(\frac{1}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{\delta(s')}{s' - s - i\epsilon} ds' \right), \tag{10}$$

where s is the B mass squared taken as a dispersion variable, and $P(s)$ is a polynomial arising from zeros of $M(s)$. $P(s)$ is related to the factorization amplitude since it contains no FSI. Therefore the FSI enhancement factor of the factorization amplitude is given by[a]

$$E = \exp\left(\frac{\mathcal{P}}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{\delta(s')}{s' - m_B^2} ds' \right). \tag{11}$$

This form of enhancement was actually proposed and studied for elastic rescattering at low energies in the late 1950's.[11] The enhancement E includes the FSI of both SD and LD. While the low energy portion of the phase integral is purely LD for an obvious reason, the high energy portion is not entirely SD.

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[a] For $s' > (m_{B^*} + m_\pi)^2$, $\delta(s')$ contains the initial-state interaction or dissociation, part of which enters the form factor of the factorization amplitude. It is understood that the initial-state interaction contributing only to the form factor has been removed from $\delta(s')$ of Eq.(11).

6.2   Numerical exercise

I first illustrate the enhancement factor with the decay $K \to \pi\pi$. Since the FSI phase is equal to the scattering phase shift below the inelastic threshold, we can evaluate the low-energy portion of the phase integral up to $\simeq 1$ GeV with the $\pi\pi$ phase shifts. The result is $E = 2.2$ and 0.8 for $I = 0$ and $I = 2$ channels, respectively, which amounts to the relative enhancement of $M_{\Delta I = 1/2}/M_{\Delta I = 3/2} = 2.9$. This is a LD enhancement independent of $\mathcal{O}_i$. Above the inelastic threshold, $\delta(s')$ is not longer equal to the phase shift and depends on decay operators. Therefore the integral above 1 GeV provides an operator-dependent enhancement similar to the SD enhancement of RG, but not identical to it.

Let us turn to the B decay.

In the two-body intermediate approximation, the FSI phase approaches $\pm 90°$ at high energies. Such an amplitude potentially acquires a dangerously large enhancement or suppression since the phase integral can be large. If the factorization gives a right ballpark value for a decay amplitude, $\delta(s')$ must not stay too large at high energies. In the random approximation, the sign of $\delta(s')$ may fluctuate with s'. In this case, the effect of enhancement and suppression would be much smaller.

Here I present a numerical estimate for $B \to K\pi(\pi\pi)$ with one model FSI phase motivated by experiment. Knowing that the phases are small for $D \to \overline{K}\pi$ but possibly small for $B \to K\pi$, I choose the FSI phase as shown in Fig.2: $\delta(s')$ rises to large values ($\sim 90°$) around 2 GeV and falls linearly to zero at $E_{\max} = O(m_B)$.[b] This $\delta(s')$ suppresses the decay amplitude since
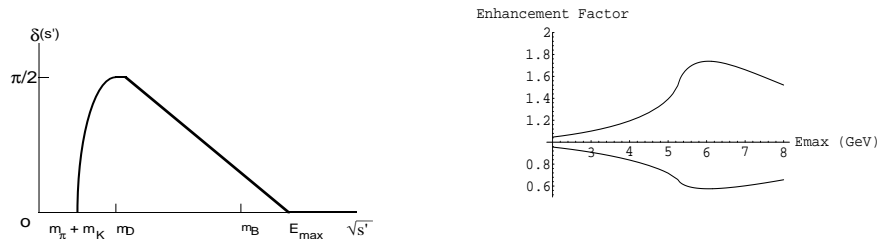

Figure 2: The model FSI phase and the enhancement (suppression) factor E vs E_{max}. The lower curve ($E < 1$) for $\delta(s') > 0$ shown here, and the upper curve for $\delta(s') \to -\delta(s')$.

the support of the phase integral is mostly below m_B where $1/(s' - m_B^2)$ is

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[b]To be precise, this $\delta(s')$ includes part of the initial-state interaction above $s' = (m_{B^*} + m_\pi)^2$. See the footnote in the preceding page.

negative. The suppression factor is given by the lower curve in the right of Fig.2. It can easily a 10∼40% correction to the factorization as $E_{\max}$ varies.

We learn two lessons from this exercise. Unlike the phase, the magnitude of amplitude receives the FSI effect from all energies. Even if the FSI phase happens to be zero at m_B, the FSI can generate a substantial correction to the factorization amplitude. Next, the low-energy (LD) contribution is power-suppressed in $1/m_B^2$, but not necessarily like $\Lambda_{\rm QCD}^2/m_B^2$. It may be more like m_D^2/m_B^2 or $m_{J/\psi}^2/m_B^2$ $(= 15 \sim 30\%)$.

7 Conclusion

If LD physics is important in two-body B decays, computing individual FSI phases is nearly an impossible proposition. What we can predict at best is their statistically likely values.

The phase and the magnitude of amplitude are related by analyticity. Multiplying a large FSI phase on a factorization amplitude is very likely inconsistent with analyticity. If an amplitude is significantly enhanced or suppressed relative to its factorization value, it indicates that the FSI phase is large and LD physics is important. When we attempt to extract the CP phases from experiment, we should be apprehensive about the level of accuracy of factorization amplitudes. We must not use calculated values of FSI phases in analysis.

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